LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

**B.Sc.** DEGREE EXAMINATION – **STATISTICS**

FIRST SEMESTER – APRIL 2011

# ST 1503/ST 1501 - PROBABILITY AND RANDOM VARIABLES

 Date : 06-04-2011 Dept. No. Max. : 100 Marks

 Time : 1:00 - 4:00

**Section A Answer ALL the questions 10x2=20**

1. Define mutually Exclusive events with an example.

2. Mention any two limitations of classical definition of probability.

3. What is the probability that a leap year selected at random will have 53 Sundays?

4. When do we say that an event A is statistically independent with respect to an event B?

 Does this mean that event B is statistically independent of the event A?

5. If A and B are independent events, Show that **Ac** and **B** are independent events.

6. What is the importance of the Baye’s Theorem?

7. Explain the term Bernoulli trials.

8. If X is a continuous random variable with probability density function f(*x*) = *k* *x* (1-*x*),

 if 0 ≤ *x* ≤ 1 and 0, otherwise, find the value of k. Hence find E (x).

9. If X and Y are two random variables such that X ≤ Y, show that

 E(X) ≤ E(Y) provided they exist.

10. State any two properties of variance of a random variable.

**Section B Answer any FIVE questions 5x8=40**

11. Define Probability Generating Function. (PGF). Obtain the PGF of a random variable X that

 follows a Poisson distribution with parameter, λ. Hence find the mean and variance of X.

12. A box contains 6 red, 4 white and 5 black balls. A person draws 4 balls from the box at

 random. Find the probability that among the balls drawn, there is at least one ball of each colour.

13. Using axioms of probability show that, for any two events A and

 B, P (A∪B) + P (A∩B) = P (A)+P (B)

14. From a vessel containing 3 white and 5 black balls, 4 balls are transferred into an empty vessel.

 From this vessel a ball is drawn and found to be white. What is the probability that, out of four

 balls transferred, 3 were white and 1 was black?

15. A bag contains 10 gold and 8 silver coins. Two successive drawings of 4 coins are made such

 that (i) coins are replaced before the second trial, (ii) the coins are not replaced before the

 second trial. Find the probability that the first drawing will give 4 gold and the second 4 silver

 coins.

16. Sixty per cent of the employees of the XYZ Corporation are college graduates. Of these, ten per

 cent are in sales. Of the employees who did not graduate from the college, eighty per cent are

 in sales.

1. What is the probability that an employee selected at random is in sales? (b) What is the

 probability that an employee selected at random is neither in sales nor a college graduate?

17. The odds against Manager X settling a wage dispute with the workers are 8: 6. The odds in

 favour of Manager Y settling the same dispute are 14:16. (a) What is the probability that

 neither settles the dispute, if they both try independently of each other? (b) What is the

 probability that the dispute will be solved?

18. If X is a continuous random variable with probability density function, f(*x*) = 1 if 0 < *x* < 1 and

 0, otherwise, Use Chebyshev’s Inequality to obtain an upper bound for P [ |X –E(X) | > 2σ ].

 Compare it with the exact probability.

**Section C Answer any TWO questions 2x20=40**

19. (a) If A, B and C are mutually independent show that A∪B and C are independent

 (b) An Urn contains four tickets marked with numbers, 112, 121, 211, 222, and one ticket is

 drawn at random. Let Ai (i=1, 2, 3, ) be the event that ith digit of the number of the ticket

 drawn is 1. Discuss the mutual independence and pair-wise independence of the events.

20. (a) State and prove Bayes theorem for future events.

 (b) Suppose that Urn I contains 1 white, 2 black and 3 red balls. Urn II contains 2 white, I black

 and 1 red balls. Urn III contains, 4 white, 5 black and 3 red balls. One Urn is chosen at

 random and two balls are drawn from it. They happen to be white and red. What is the

 probability that they have come from I, II or III?

21. (a) State and prove Chebyshev’s inequality. Bring out its importance.

(b) A petrol pump is supplied with petrol once in a day. If its daily volume of sales (X) (in

 thousands of litres) is distributed by

 f(*x*) = *5* (1-*x*) 4, if 0 ≤ *x* ≤ 1

 What must the capacity be of is tank in order that the probability that its supply will get

 exhausted in a given day shall be 0.01?

22. (a) For any two events A. B and C, show that

 P (A∪B⎟ C ) = P (A⎟ C) + P (B⎟ C) − P (A∩B⎟ C)

and P (A∩BC⎟ C ) + P (A∩B⎟ C) = P (A⎟ C).

1. Two computers A and B are to be marketed. A salesman who was assigned the job of finding customers for them has 60% chance of succeeding in the case of computer A and 40% chance of succeeding in the case of Computer B. The two computers can be sold independently. Given that he was able to sell at least one computer, what is the probability that computer A has been sold.

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